كلية الهندسة
جامعة أسوان

Water Supply Engineering

Code: CONE 323

Lecture: 9

Course Instructor:

Dr. Mohamed Fekry

تابع

الفصل الثانى

Hydraulic analysis of drinking water networks

التصميم الهيدروليكي لخطوط المواسير المستخدمة

في شبكات مياه الشرب

A pipe network is analyzed for the determination of the nodal pressure heads and the link discharges.

Hardy Cross Method

The method is based on the following basic equations of continuity of flow and head loss that should be satisfied:

1. The sum of inflow and outflow at a node should be equal:

$$Q_{in} = Q_{out}$$

2. The algebraic sum of the head loss in a loop must be equal to zero:

$$\sum_{\text{loop } k} K_i Q_i |Q_i| = 0$$

Where

$$K_i = \frac{8 f_i L_i}{\pi^2 g D_i^5},$$

Calculation of correction value delta Q

$$\Delta Q_k = -\frac{\sum_{\text{loop } k} K_i Q_i |Q_i|}{2 \sum_{\text{loop } k} K_i |Q_i|}.$$

$$Q_{i\,new} = Q_{i\,old} + \Delta Q_k$$

The overall procedure for the looped network analysis can be summarized in the following steps:

1. Number all the nodes and pipe links. Also number the loops. For clarity, pipe numbers are circled and the loop numbers are put in square brackets.
2. Adopt a sign convention that a pipe discharge is positive if it flows from a lower node number to a higher node number, otherwise negative.
3. Apply nodal continuity equation at all the nodes to obtain pipe discharges. Starting from nodes having two pipes with unknown discharges, assume an arbitrary discharge (say 0.1 m^3/s) in one of the pipes and apply continuity equation (3.13) to obtain discharge in the other pipe. Repeat the procedure until all the pipe flows are known. If there exist more than two pipes having unknown discharges, assume arbitrary discharges in all the pipes except one and apply continuity equation to get discharge in the other pipe. The total number of pipes having arbitrary discharges should be equal to the total number of primary loops in the network.

4. Assume friction factors $f_i = 0.02$ in all pipe links and compute corresponding K_i using Eq. (3.15). However, f_i can be calculated iteratively using Eq. (2.6a).
5. Assume loop pipe flow sign convention to apply loop discharge corrections; generally, clockwise flows positive and counterclockwise flows negative are considered.
6. Calculate ΔQ_k for the existing pipe flows and apply pipe corrections algebraically.
7. Apply the similar procedure in all the loops of a pipe network.

Repeat steps 6 and 7 until the discharge corrections in all the loops are relatively very small.

$\Delta Q < 3$ % of max. Q inside the loop

Example 3.3. A single looped network as shown in Fig. 3.10 has to be analyzed by the Hardy Cross method for given inflow and outflow discharges. The pipe diameters *D* and lengths *L* are shown in the figure. Use Darcy–Weisbach head loss–discharge relationship assuming a constant friction factor $f = 0.02$.

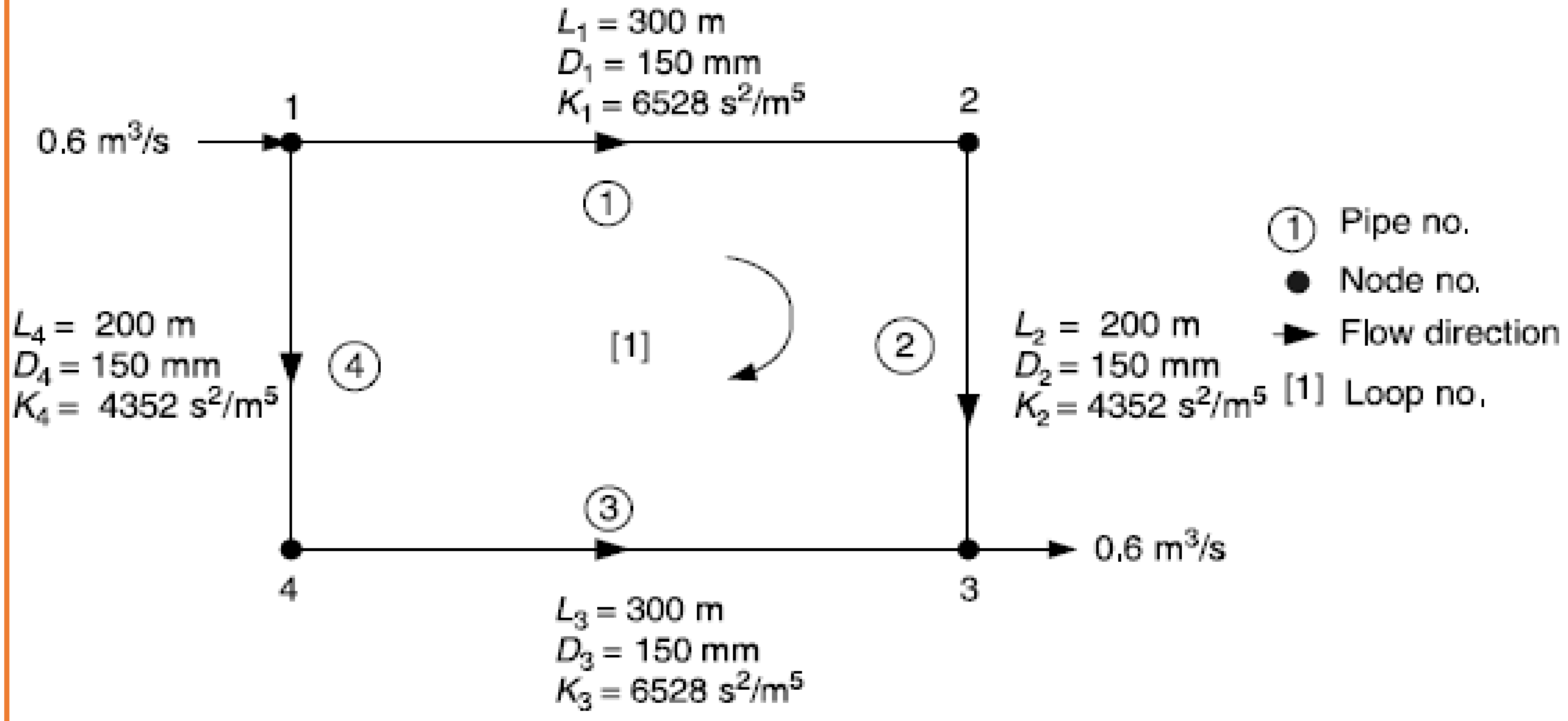


Figure 3.10. Single looped network.

Step 1: The pipes, nodes, and loop are numbered as shown in Fig. 3.10.

Step 2: Adopt the following sign conventions:
A positive pipe discharge flows from a lower node to a higher node.
Inflow into a node is positive withdrawal negative.

Step 3: Apply continuity equation to obtain pipe discharges.

$$Q_1 + Q_4 = q_1 \text{ or } Q_4 = q_1 - Q_1, \text{ hence } Q_4 = 0.6 - 0.1 = 0.5\,\text{m}^3/\text{s}.$$

$$-Q_1 + Q_2 = q_2 \text{ or } Q_2 = q_2 + Q_1, \text{ hence } Q_2 = 0 + 0.1 = 0.1\,\text{m}^3/\text{s}.$$

Step 4: For assumed pipe friction factors $f_i = 0.02$, the calculated K values as $K = 8fL/\pi^2 gD^5$ for all the pipes are given in the Fig. 3.10.

Step 5: Adopted clockwise flows in pipes positive and counterclockwise flows negative.

Step 6: The discharge correction for the initially assumed pipe discharges can be calculated as follows:

Iteration 1

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
1	0.10	6528.93	65.29	1305.79	0.30
2	0.10	4352.62	43.53	870.52	0.30
3	−0.50	6528.93	−1632.23	6528.93	−0.30
4	−0.50	4352.62	−1088.15	4352.62	−0.30
Total			−2611.57	13,057.85	
ΔQ			$-(-2611.57/13{,}057) = 0.20\ m^3/s$		

Repeat the process again for the revised pipe discharges as the discharge correction is quite large in comparison to pipe flows:

Iteration 2

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
1	0.30	6528.93	587.60	3917.36	0.30
2	0.30	4352.62	391.74	2611.57	0.30
3	−0.30	6528.93	−587.60	3917.36	−0.30
4	−0.30	4352.62	−391.74	2611.57	−0.30
Total			0.00	13,057.85	
ΔQ			$= -(0/13{,}057) = 0.00\ m^3/s$		

As the discharge correction $\Delta Q = 0$, the final discharges are

$$Q_1 = 0.3\ \mathrm{m^3/s}$$
$$Q_2 = 0.3\ \mathrm{m^3/s}$$
$$Q_3 = 0.3\ \mathrm{m^3/s}$$
$$Q_4 = 0.3\ \mathrm{m^3/s}.$$

Solving the same problem by help of EXCEL

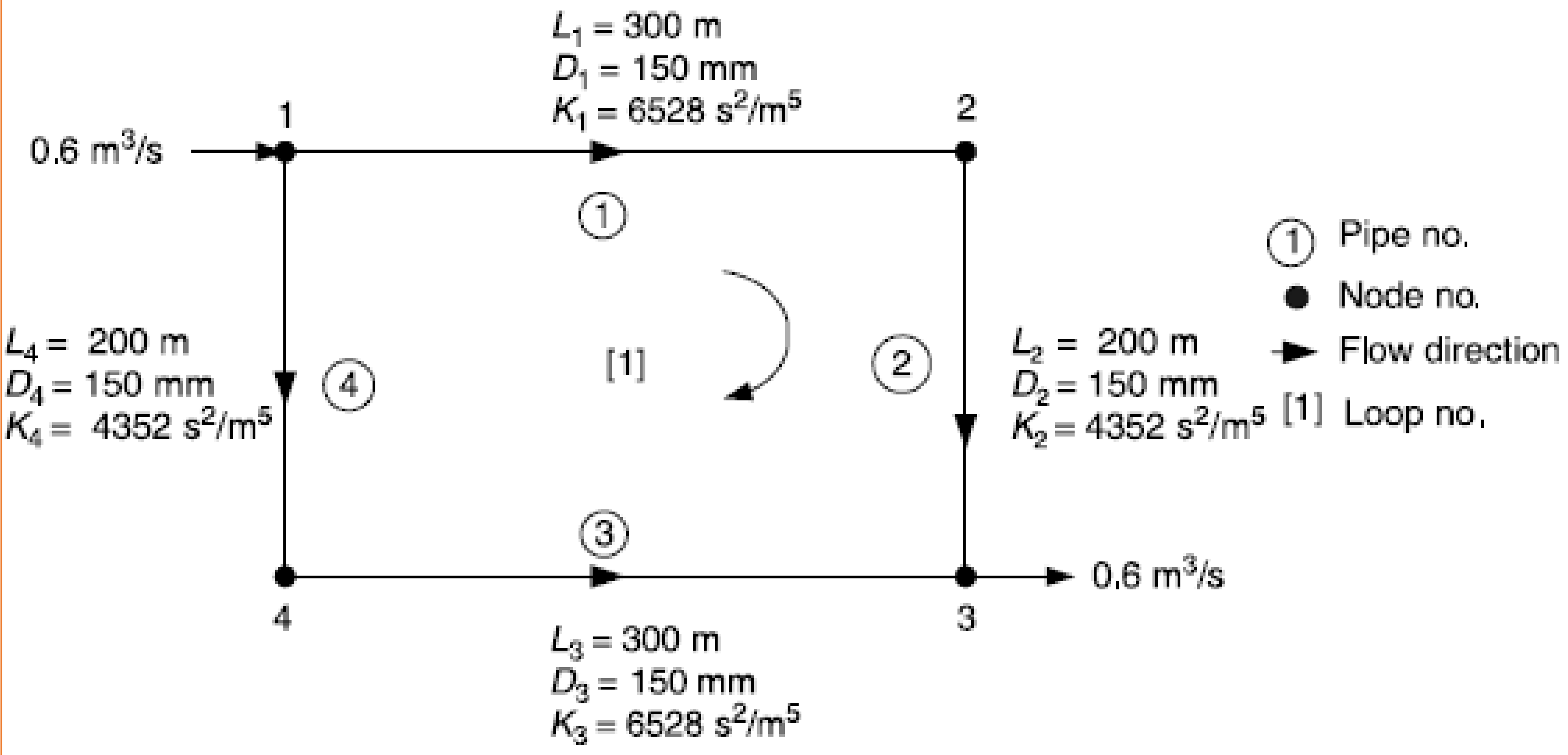
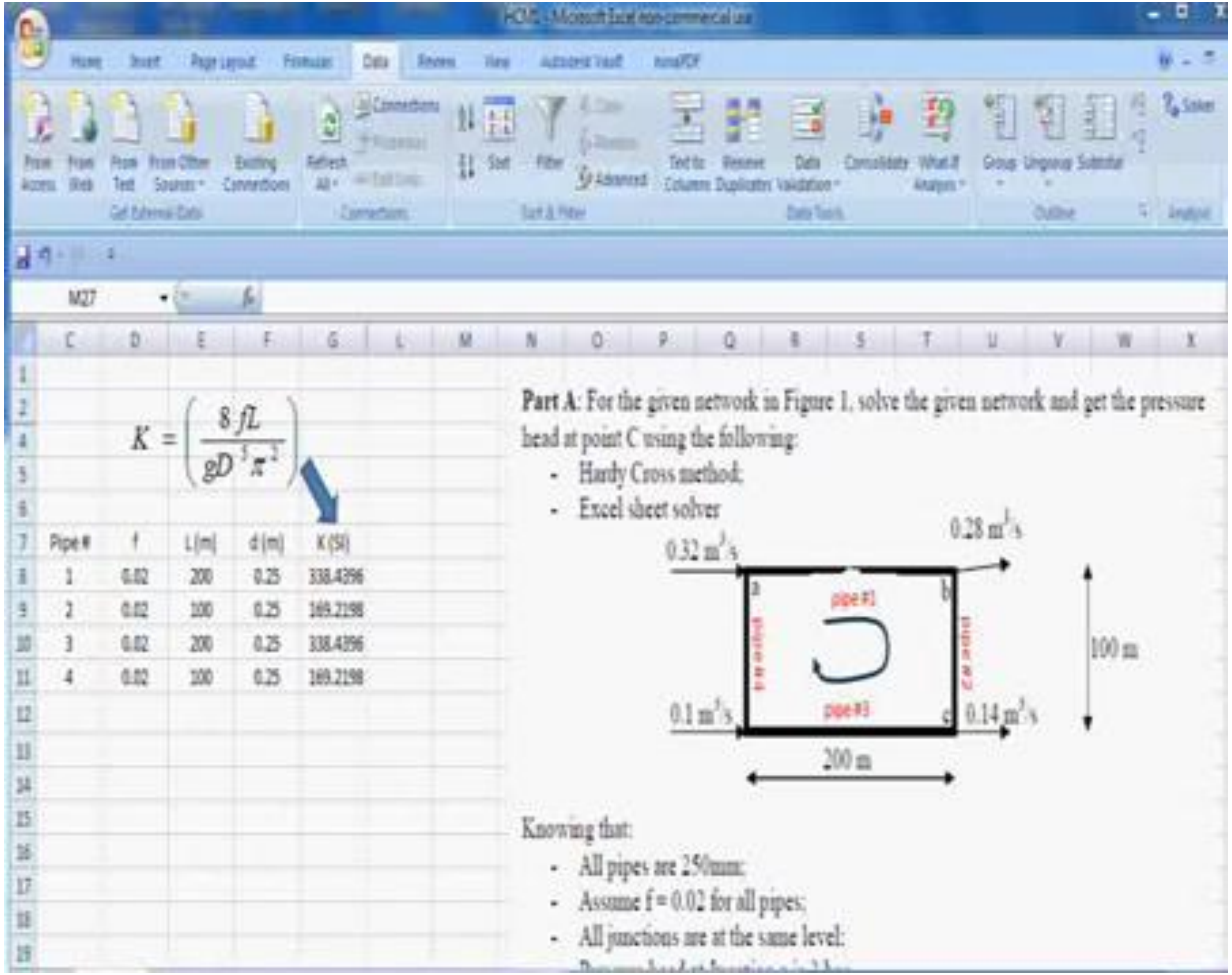


Figure 3.10. Single looped network.



$$K = \left(\frac{8\,fL}{gD^5\pi^2} \right)$$

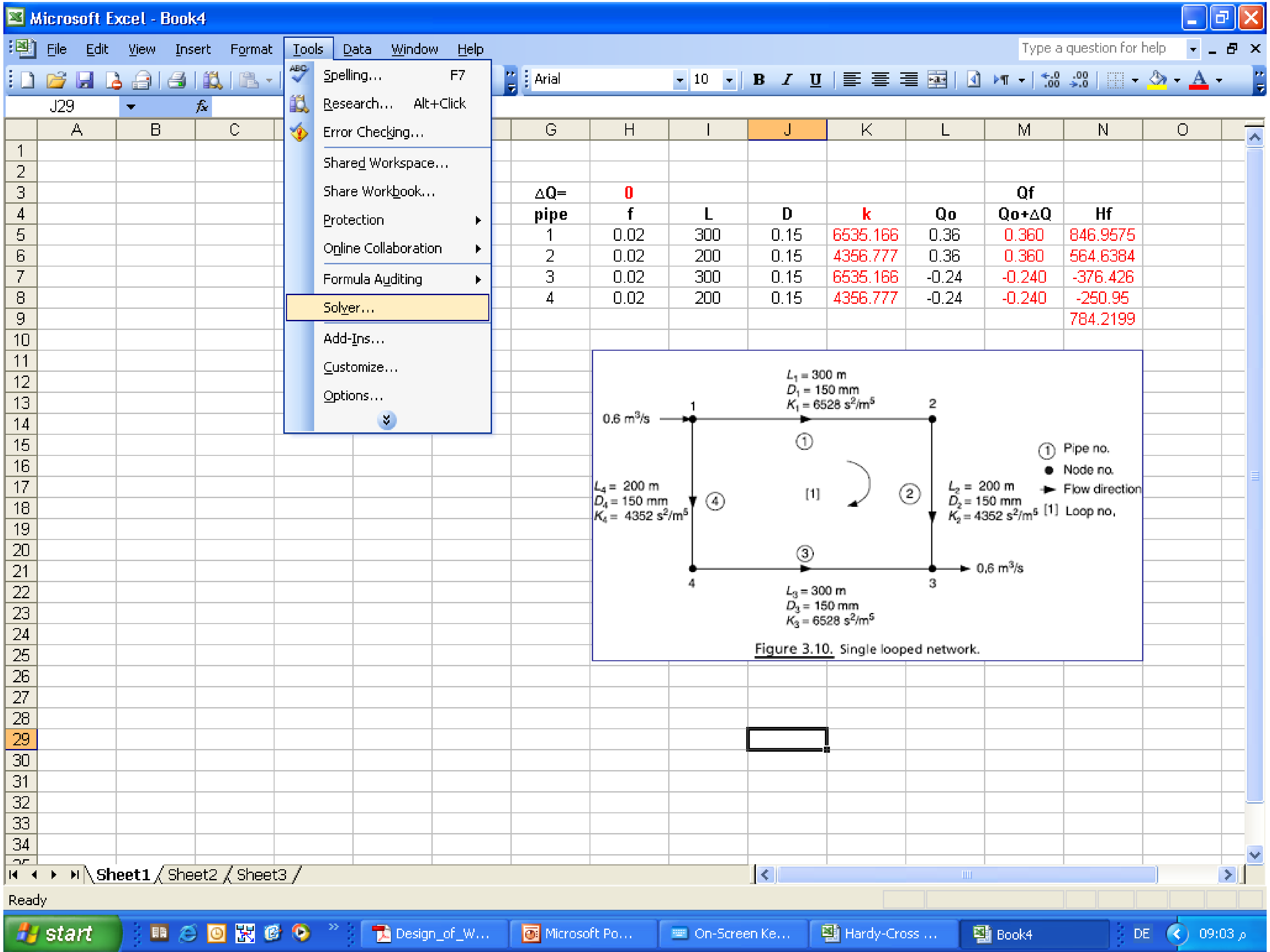
Pipe #	f	L (m)	d (m)	K (SI)
1	0.02	200	0.25	338.4396
2	0.02	100	0.25	169.2198
3	0.02	200	0.25	338.4396
4	0.02	100	0.25	169.2198

Part A: For the given network in Figure 1, solve the given network and get the pressure head at point C using the following:

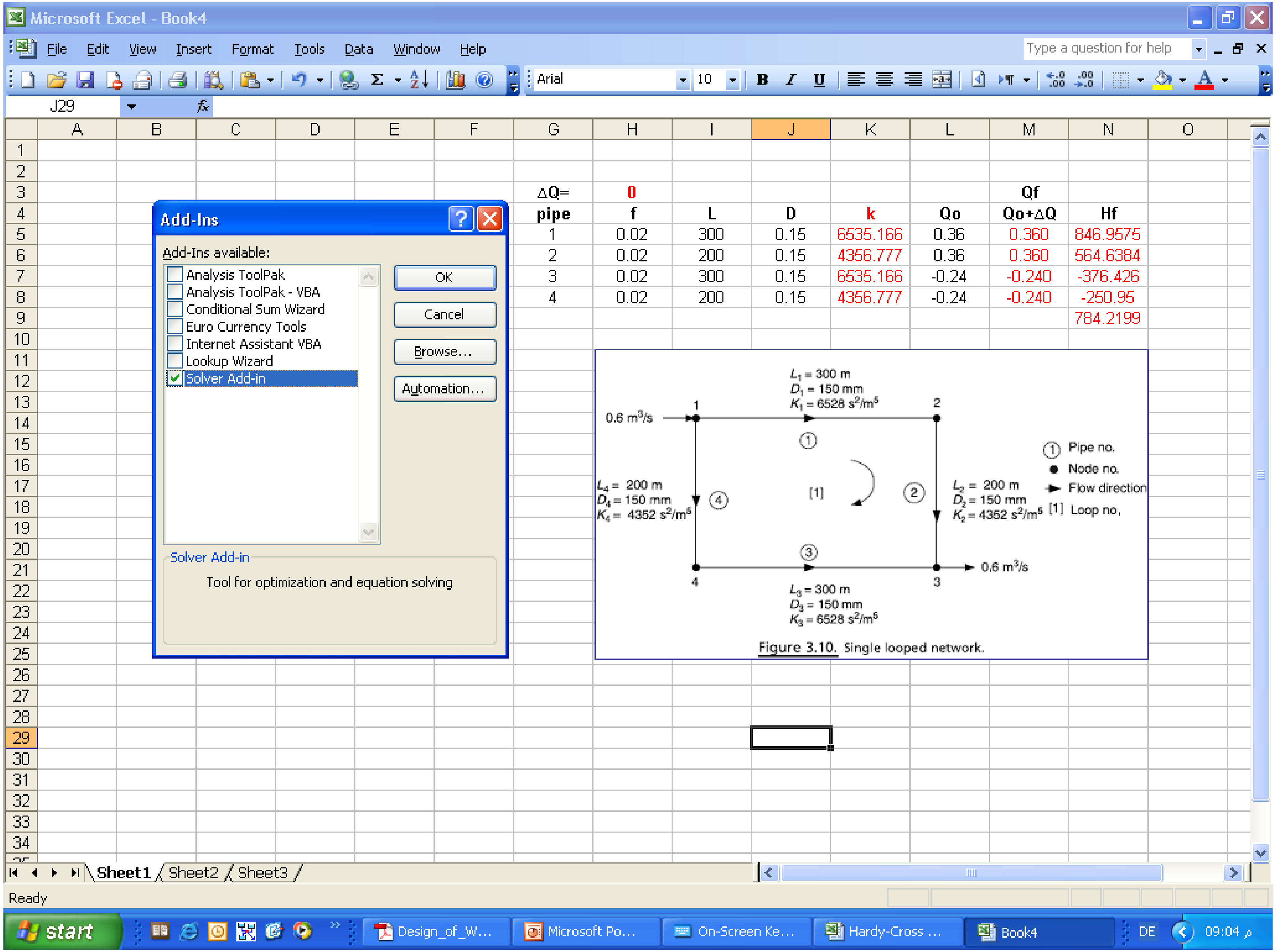
- Hardy Cross method;
- Excel sheet solver

Knowing that:

- All pipes are 250mm;
- Assume f = 0.02 for all pipes;
- All junctions are at the same level;



ΔQ=	0					Qf	
pipe	f	L	D	k	Qo	Qo+ΔQ	Hf
1	0.02	300	0.15	6535.166	0.36	0.360	846.9575
2	0.02	200	0.15	4356.777	0.36	0.360	564.6384
3	0.02	300	0.15	6535.166	-0.24	-0.240	-376.426
4	0.02	200	0.15	4356.777	-0.24	-0.240	-250.95
							784.2199


Microsoft Excel - Book4
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Type a question for help
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10
J29
ΔQ= 0
pipe f L D k Qo Qf Qo+ΔQ Hf
1 0.02 300 0.15 6535.166 0.36 0.360 846.9575
2 0.02 200 0.15 4356.777 0.36 0.360 564.6384
3 0.02 300 0.15 6535.166 -0.24 -0.240 -376.426
4 0.02 200 0.15 4356.777 -0.24 -0.240 -250.95
784.2199
Add-Ins
Add-Ins available:
Analysis ToolPak
Analysis ToolPak - VBA
Conditional Sum Wizard
Euro Currency Tools
Internet Assistant VBA
Lookup Wizard
Solver Add-in
OK
Cancel
Browse...
Automation...
Solver Add-in
Tool for optimization and equation solving
L1 = 300 m
D1 = 150 mm
K1 = 6528 s²/m⁵
0.6 m³/s
L4 = 200 m
D4 = 150 mm
K4 = 4352 s²/m⁵
L2 = 200 m
D2 = 150 mm
K2 = 4352 s²/m⁵
L3 = 300 m
D3 = 150 mm
K3 = 6528 s²/m⁵
0.6 m³/s
Pipe no.
Node no.
Flow direction
Loop no.
Figure 3.10. Single looped network.
Sheet1 Sheet2 Sheet3
Ready
start
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On-Screen Ke...
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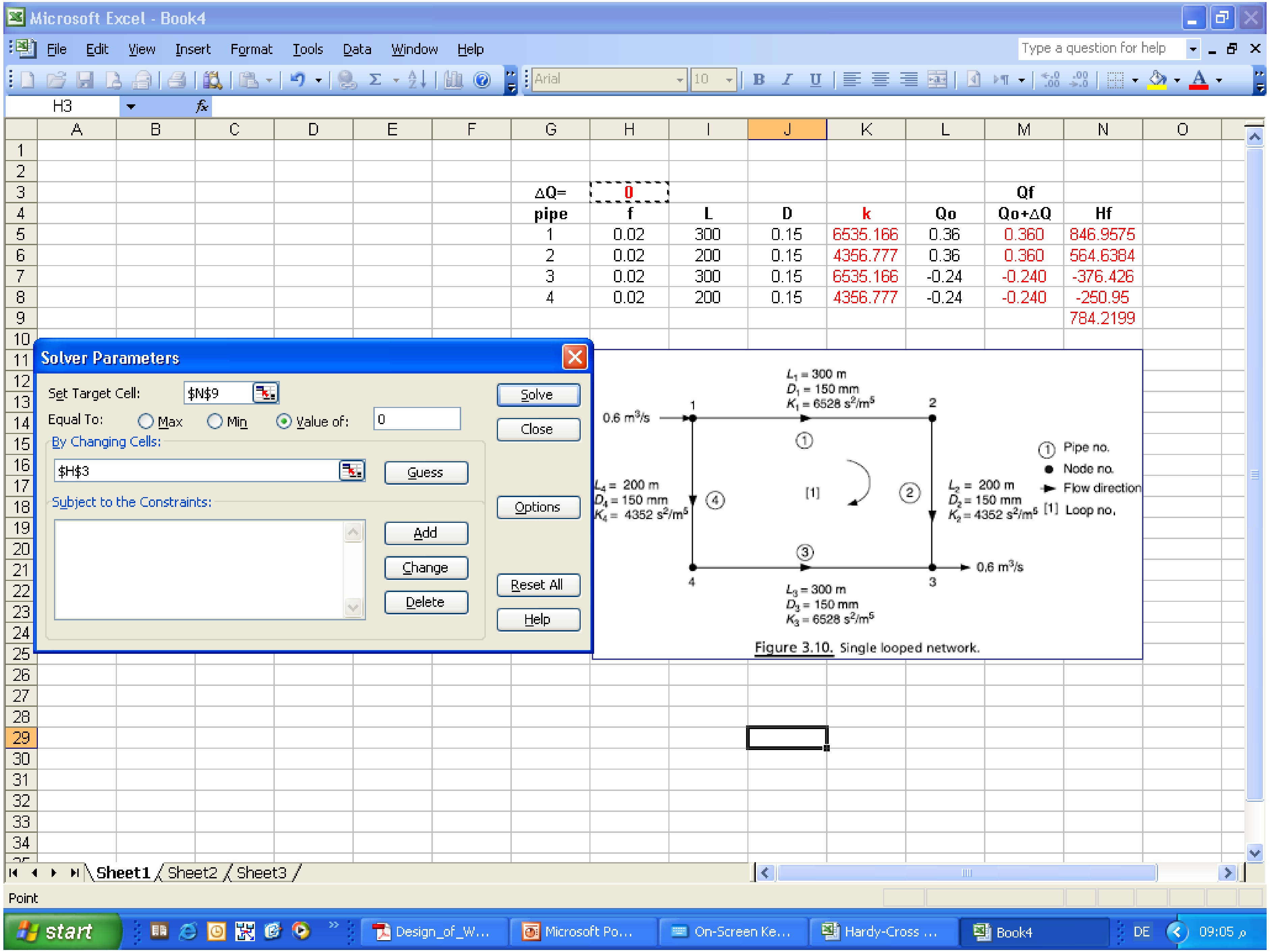
Microsoft Excel - Book4
File Edit View Insert Format Tools Data Window Help
H3

pipe	f	L	D	k	Qo	Qf Qo+ΔQ	Hf
ΔQ=	0						
1	0.02	300	0.15	6535.166	0.36	0.360	846.9575
2	0.02	200	0.15	4356.777	0.36	0.360	564.6384
3	0.02	300	0.15	6535.166	-0.24	-0.240	-376.426
4	0.02	200	0.15	4356.777	-0.24	-0.240	-250.95
							784.2199

Solver Parameters
Set Target Cell: N9
Equal To: Max Min Value of: 0
By Changing Cells:
H3
Subject to the Constraints:
Solve
Close
Guess
Options
Add
Change
Delete
Reset All
Help
L_1 = 300 m
D_1 = 150 mm
K_1 = 6528 s²/m⁵
0.6 m³/s
L_4 = 200 m
D_4 = 150 mm
K_4 = 4352 s²/m⁵
L_2 = 200 m
D_2 = 150 mm
K_2 = 4352 s²/m⁵
L_3 = 300 m
D_3 = 150 mm
K_3 = 6528 s²/m⁵
0.6 m³/s
[1]
Pipe no.
Node no.
Flow direction
[1] Loop no.
Figure 3.10. Single looped network.
Sheet1 Sheet2 Sheet3
Point
start
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DE 09:05

Microsoft Excel - Book4

ΔQ=	-0.06					Qf	
pipe	f	L	D	k	Qo	Qo+ΔQ	Hf
1	0.02	300	0.15	6535.166	0.36	0.300	588.165
2	0.02	200	0.15	4356.777	0.36	0.300	392.11
3	0.02	300	0.15	6535.166	-0.24	-0.300	-588.165
4	0.02	200	0.15	4356.777	-0.24	-0.300	-392.11
							0

Solver Results

Solver found a solution. All constraints and optimality conditions are satisfied.

Reports: Answer, Sensitivity, Limits

- Keep Solver Solution
- Restore Original Values

OK | Cancel | Save Scenario... | Help

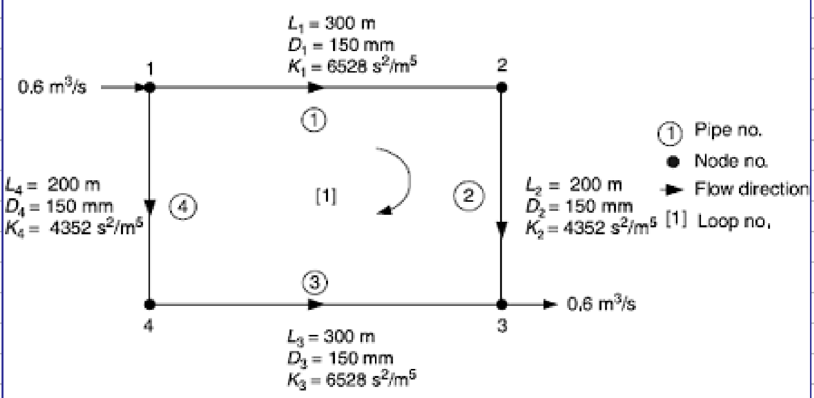
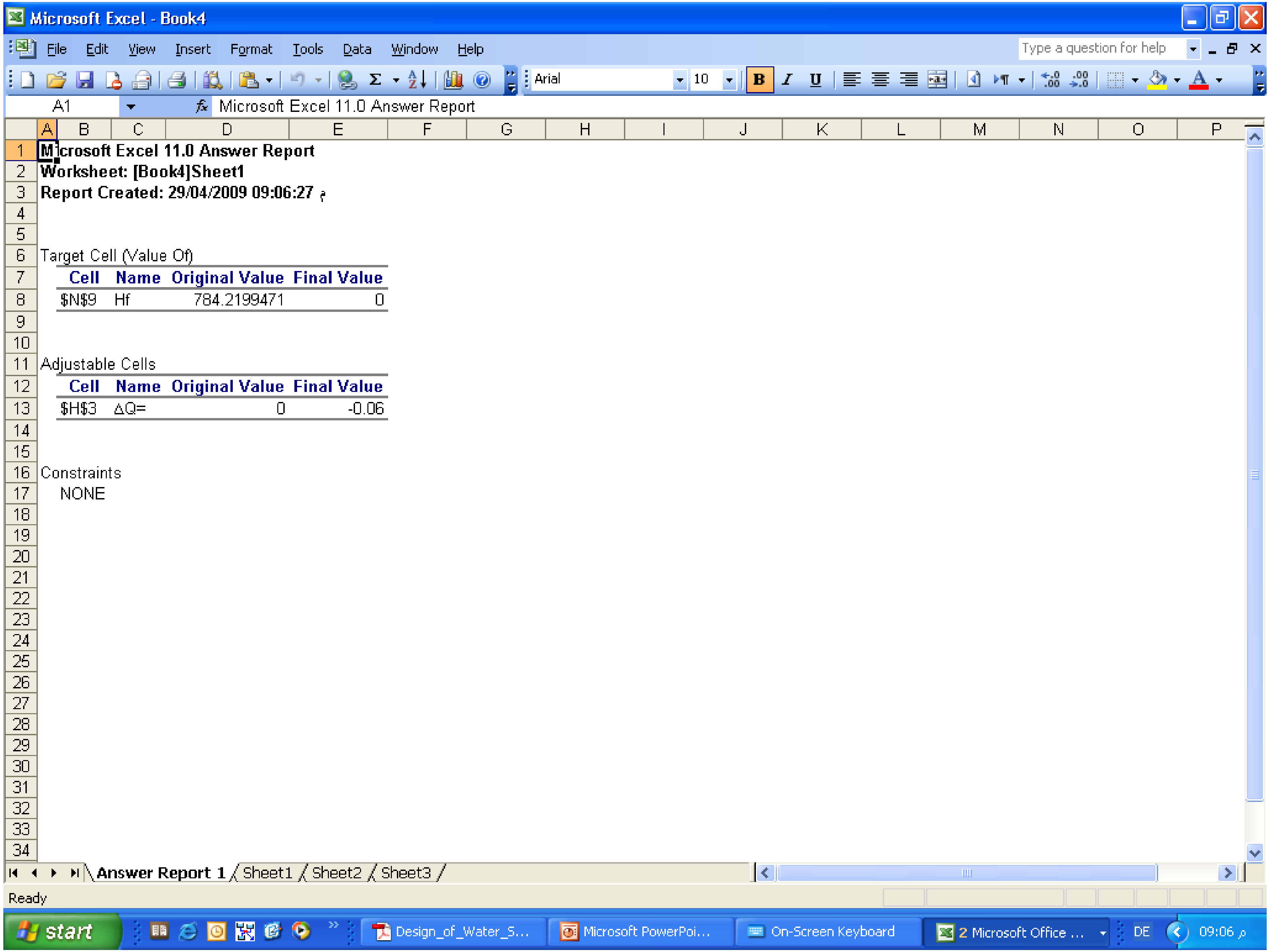


Figure 3.10. Single looped network.



Microsoft Excel 11.0 Answer Report
Worksheet: [Book4]Sheet1
Report Created: 29/04/2009 09:06:27 م

Target Cell (Value Of)

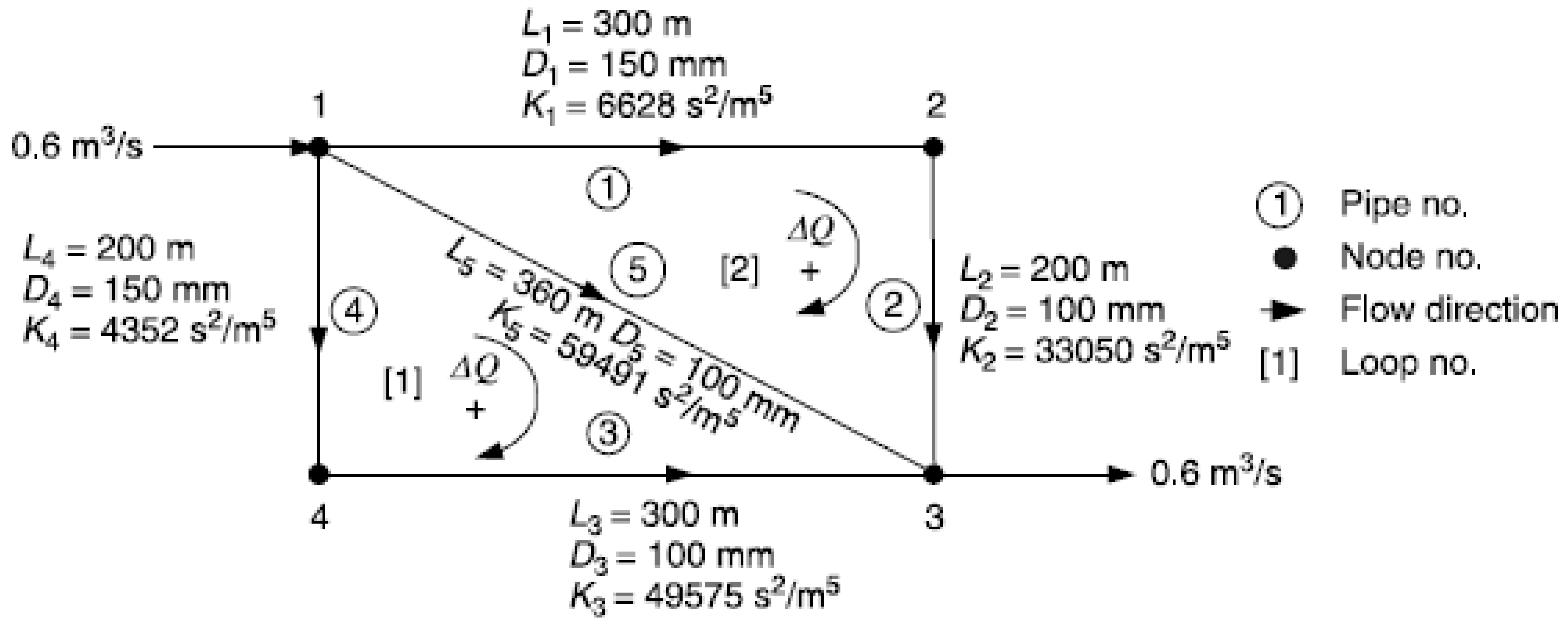
Cell	Name	Original Value	Final Value
N9	Hf	784.2199471	0

Adjustable Cells

Cell	Name	Original Value	Final Value
H3	ΔQ=	0	-0.06

Constraints
NONE

Example 3.4. The pipe network of two loops as shown in Fig. 3.11 has to be analyzed by the Hardy Cross method for pipe flows for given pipe lengths L and pipe diameters D. The nodal inflow at node 1 and nodal outflow at node 3 are shown in the figure. Assume a constant friction factor $f = 0.02$.



To apply continuity equation for initial pipe discharges, the discharges in pipes 1 and 5 equal to 0.1 m^3/s are assumed. The obtained discharges are

$Q_1 = 0.1\,m^3/s$ (flow from node 1 to node 2)
$Q_2 = 0.1\,m^3/s$ (flow from node 2 to node 3)
$Q_3 = 0.4\,m^3/s$ (flow from node 4 to node 3)
$Q_4 = 0.4\,m^3/s$ (flow from node 1 to node 4)
$Q_5 = 0.1\,m^3/s$ (flow from node 1 to node 3)

The discharge correction ΔQ is applied in one loop at a time until the ΔQ is very small in all the loops. ΔQ in Loop 1 (loop pipes 3, 4, and 5) and corrected pipe discharges are given in the following table:

Loop 1: Iteration 1

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
3	−0.40	49,576.12	−7932.18	39,660.89	−0.25
4	−0.40	4352.36	−696.38	3481.89	−0.25
5	0.10	59,491.34	594.91	11,898.27	0.25
Total			−8033.64	55,041.05	
ΔQ			0.15 m^3/s		

Thus the discharge correction ΔQ in loop 1 is 0.15 m^3/s. The discharges in loop pipes are corrected as shown in the above table. Applying the same methodology for calculating ΔQ for Loop 2:

Loop 2: Iteration 1

Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
1	0.10	6528.54	65.29	1305.71	0.19
2	0.10	33,050.74	330.51	6610.15	0.19
5	−0.25	59,491.34	−3598.93	29,264.66	−0.16
Total			−3203.14	37,180.52	
ΔQ			0.09 m^3/s		

The process of discharge correction is in repeated until the ΔQ value is very small as shown in the following tables:

Loop 1: Iteration 5

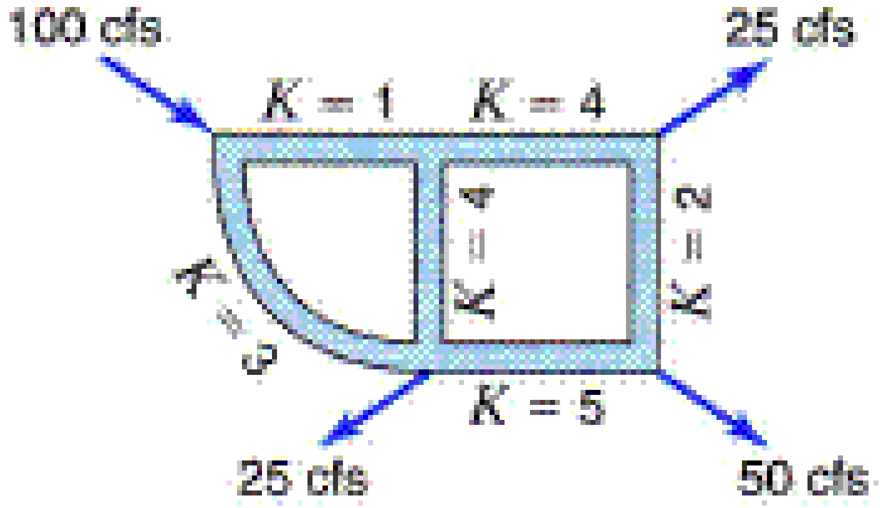
Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
3	−0.193	49,576.12	−1840.21	19,102.92	−0.192
4	−0.193	4352.36	−161.55	1677.07	−0.192
5	0.181	59,491.34	1954.67	21,567.21	0.182
Total			−47.09	42,347.21	
ΔQ			0.001 m^3/s		

Loop 2: Iteration 5

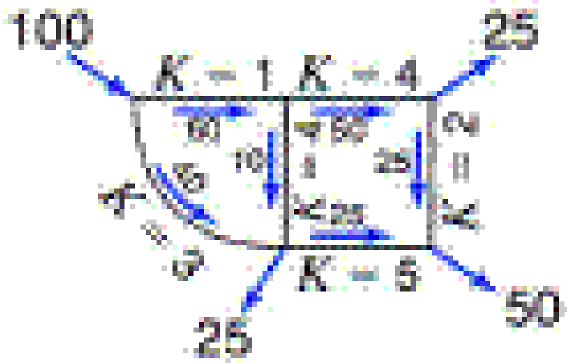
Pipe	Flow in Pipe Q (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
1	0.222	6528.54	322.40	2901.61	0.223
2	0.222	33,050.74	1632.17	14,689.40	0.223
5	-0.182	59,491.34	−1978.73	21,699.52	−0.182
Total			−24.15	39,290.53	
ΔQ			0.001 m^3/s		

Numerical Example II

Apply Hardy Cross Method (n=2):



Numerical Example II



First approximation

Left loop

$h_L = KQ_0^n$	$n\lvert KQ_0^{n-1}\rvert$
$1 \times 60^2 = 3{,}600$	$1 \times 2 \times 60 = 120$
$4 \times 10^2 = 400$	$4 \times 2 \times 10 = 80$
$3 \times 40^2 = 4{,}800$	$3 \times 2 \times 40 = 240$
800	440

$$\Delta Q_1 = \frac{-(-800)}{440} = 2$$

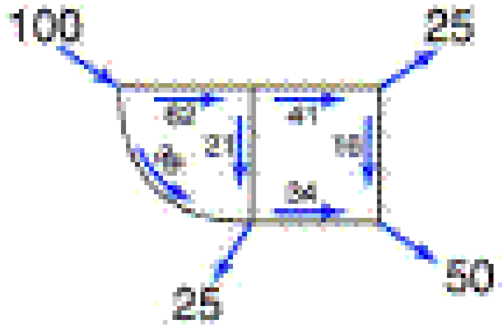
$$\Delta Q = -\frac{\sum KQ_0^2}{2\sum KQ_0}$$

Right loop

$h_L = KQ_0^n$	$n\lvert KQ_0^{n-1}\rvert$
$4 \times 50^2 = 10{,}000$	$4 \times 2 \times 50 = 400$
$2 \times 25^2 = 1{,}250$	$2 \times 2 \times 25 = 100$
$4 \times 10^2 = 400$	$4 \times 2 \times 10 = 80$
$5 \times 25^2 = 3{,}125$	$5 \times 2 \times 25 = 250$
$7{,}725$	830

$$\Delta Q_1 = \frac{-(+7725)}{830} = 9$$

Numerical Example II



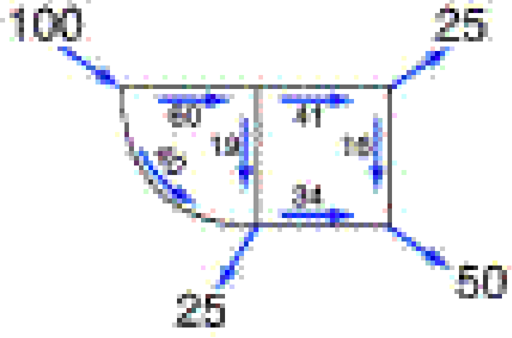
After first correction

$1 \times 62^2 = 3{,}844$ | $1 \times 2 \times 62 = 124$
$4 \times 21^2 = 1{,}764$ | $4 \times 2 \times 21 = 168$
$3 \times 38^2 = 4{,}332$ | $3 \times 2 \times 38 = 228$
$1{,}276$ | 520

$$\Delta Q_2 = \frac{-(+1276)}{520} \approx 2$$

$4 \times 41^2 = 6{,}724$ | $4 \times 2 \times 41 = 328$
$2 \times 16^2 = 512$ | $2 \times 2 \times 16 = 64$
$4 \times 21^2 = 1{,}764$ | $4 \times 2 \times 21 = 168$
$5 \times 34^2 = 5{,}780$ | $5 \times 2 \times 34 = 340$
308 | 900

$$\Delta Q_2 = \frac{-(-308)}{900} \approx 0$$



After second correction

Extended Example

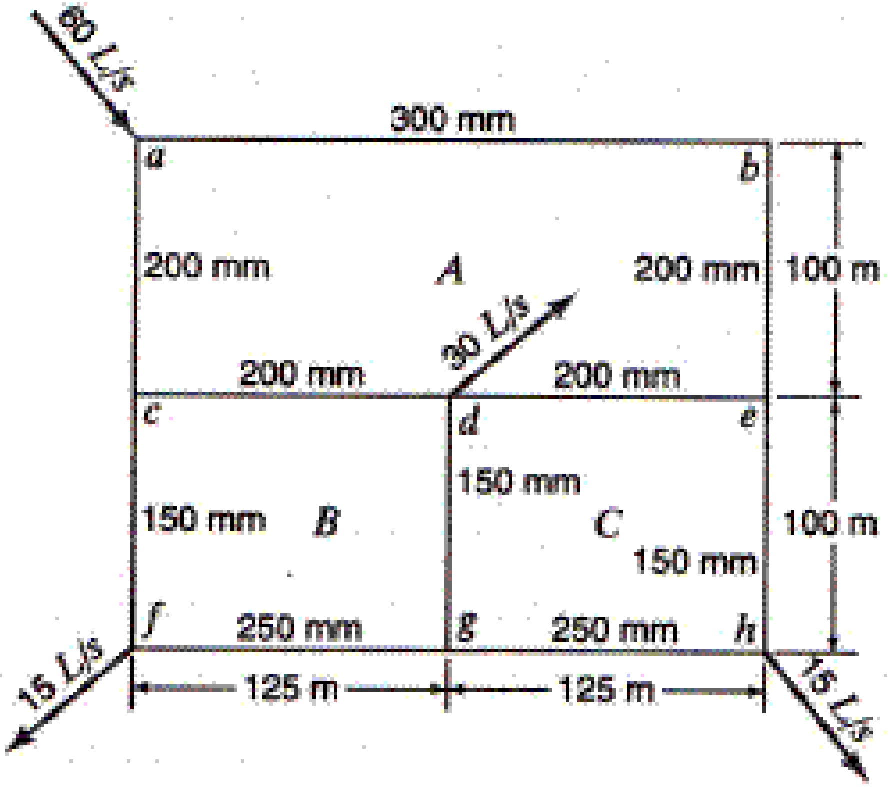


Figure. P8.119

Homework

Solver for DWDS multi-loops

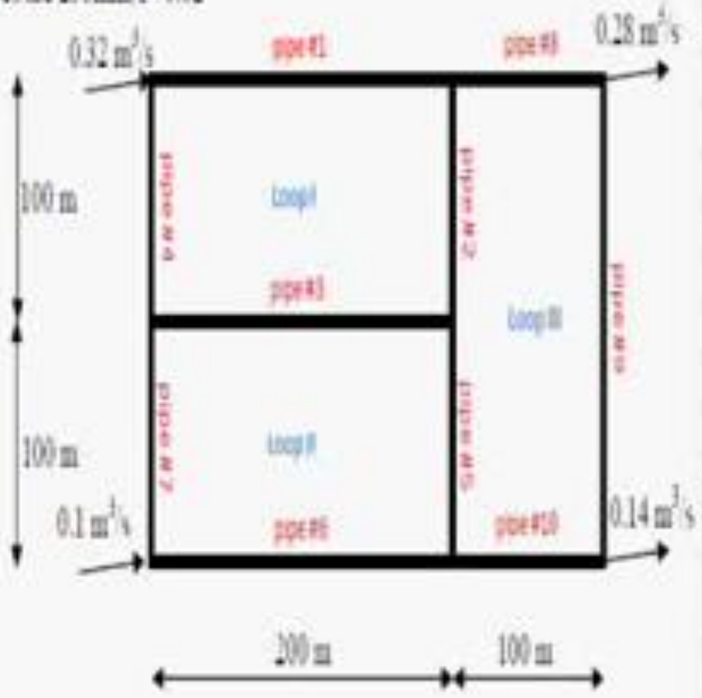
$$K = \left(\frac{8fL}{gD^5\pi^2}\right)$$

Loop	Pipe #	L(m)	D(m)	f	K
I	1	200	0.25	0.02	338.4396
	2	100	0.25	0.02	169.2198
	3	200	0.25	0.02	338.4396
	4	100	0.25	0.02	169.2198
II	3	200	0.25	0.02	338.4396
	5	100	0.25	0.02	169.2198
	6	200	0.25	0.02	338.4396
	7	100	0.25	0.02	169.2198
III	8	100	0.25	0.02	169.2198
	9	200	0.25	0.02	338.4396
	10	100	0.25	0.02	169.2198
	5	100	0.25	0.02	169.2198
	2	100	0.25	0.02	169.2198

Solve the following network using:

- Excel solver;
- Modified Hardy-Cross Jacobian Matrix Approach taken in class

All pipes are 250mm, f= 0.02



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